MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2014

09-04-2014 (Online-1)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

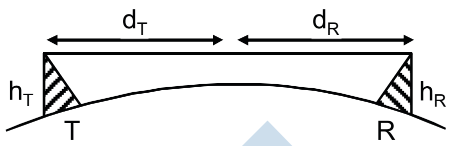
PART-A- PHYSICS

1. A transmitting antenna at the top of a tower has a height 32 m and the height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode?

(A) 45.5 km (B*) 55.4 km (C) 455 km (D) 54.5 km

Sol. $d_T = \sqrt{2Rh_T} = \sqrt{2 \times 6400 \times 10^3 \times 32}$

$= 202 \times 102 \text{ m} = 20.20 \text{ km}$



$d_R = \sqrt{2Rh_R} = \sqrt{2 \times 6400 \times 10^3 \times 50}$

$= 25.3 \text{ km}$

$\therefore$ d = $d_T + d_R$ = 20.2 + 25.3 = 45.5 km

2. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking R = 8.3 JK^{-1} $mole^{-1}$, 1 atm = 10^5 Pa and $N_{Avogadro} = 6 \times 10^{23}$ $mole^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of:

(A) 0.2 μm (B*) 0.2 mm (C) 0.2 cm (D) 0.2 nm

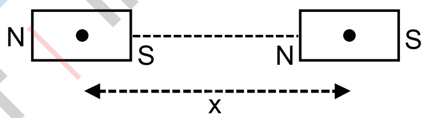
Sol. $\lambda = \frac{kt}{\sqrt{2}\pi d^2 P}$

$= \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2}\pi \times 10^{-20} \times 4 \times 10^{-10}}$

$= \frac{1.38 \times 3}{\sqrt{2} \times 4\pi} \times 10^{-9}$

= 0.2 nm

3. The mid points of two small magnetic dipoles of length d in end-on positions, are separated by a distance x, (x > > d) The force between them is proportional to x^{-n} where n is:



(A) 3 (B*) 4 (C) 2 (D) 1

Sol. $F \propto \frac{1}{r^4}$

4. Three capacitances, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of:

(A*) 6 μF (B) 2 μF (C) 1 μF (D) 4.5 μ F

Sol. When all in series

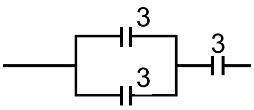
$\frac{1}{C_{eq}} = \frac{3}{3}$

C_{eq} = 1Mf (2 not possible)

when 3 is parallel

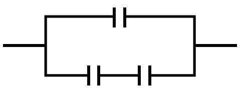
C_{eq} = 9μF

2 parallel 1 series



$C_{eq} = \frac{6 \times 3}{9} = 2\mu F$ (3 option not possible)

2 series 1 parallel



$\frac{3 \times 3}{6} + 3 = 4.5\ \mu F$

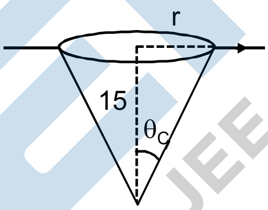
(1 option not possible)

Hence answer is (4)

5. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$ and the diver's eyes are 15 cm below the surface of water. Then the radius of the circle is:

(A*) $\frac{15 \times 3}{\sqrt{7}}$cm   (B) $\frac{15 \times \sqrt{7}}{3}$cm   (C) 15 $\times$ $3 \times \sqrt{5}$cm   (D) 15 $\times 3\sqrt{7}$cm

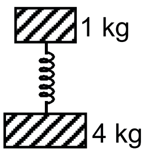
Sol. $\sin\theta_C = \frac{1}{4/3} = \frac{3}{4}$



$\tan\theta_C = \frac{3}{\sqrt{7}} = \frac{r}{h}$

$r = \frac{3}{\sqrt{7}} \times 15$

6. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude0 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take g = 10 ms^{-2}).



(A) 10 N   (B) 40 N   (C) 20 N   (D*) 60 N

Sol. T-mg = Mw^2A

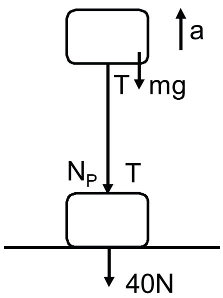
$= 1 \times 625 \times \frac{1.6}{100}$

= 10 N

T = 20 N

N = T + 40

= 60 N



7. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio $\frac{x}{y}$ is:

(A) $\left(\frac{R+d}{R-d}\right)$ (B) $\left(1-\frac{2d}{R}\right)$ (C) $\left(\frac{R-d}{R+d}\right)$ (D*) $\left(1-\frac{d}{R}\right)$

Sol. height talances additional presence hence

$\rho g_s x = \rho\, g_{depth}\, y$

$g_s\, x = g_s(1 - d/R)y$

$\frac{x}{y} = 1 - \frac{d}{R}$

8. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is:

(A*) 30.6 eV (B) 122.4 eV (C) 3.4 eV (D) 13.6 eV

Sol. B.E. = 3.4 × 9 = 3.6 eV

9. Water of volume 2 L in a closed container is heated with a coil of 1 kW. While water is heated, the container loses energy at a rate of 160 J/s. In how much time will the temperature of water rise from 27°C to 77°C? (Specific heat of water is 4.2 kJ/kg and that of the container is negligible).

(A) 14 min (B) 7 min (C) 6 min 2s (D*) 8 min 20 s

Sol. 1000 – 160 = 840 J/s

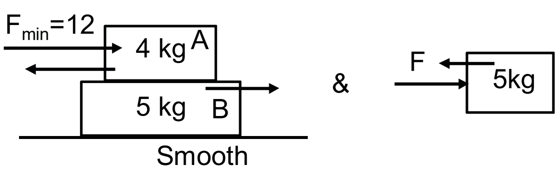
t-840 = 2 × 4.2 × 103 × 50

$t = \frac{500}{60} = 8$ min 20 s.

10. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied on B for the blocks to move together will be:

(A*) 27 N (B) 48 N (C) 30 N (D) 25 N

Sol.



12 = 9a

a = 4/3

$\boxed{5\text{ kg}} \rightarrow f = 5(4/3)$
$= 20/3$

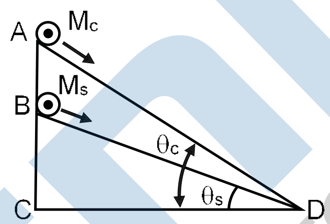
$\boxed{4\text{ kg}} \rightarrow f = 2/3 = 4a$
$a = 5/3$

F – f = 5 × 4/3

$$F - \frac{20}{3} = \frac{20}{3}$$

$$f = \frac{40}{3}$$

11. A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively. (See Figure). If they roll on the incline without slipping such that their accelerations are the same, then the ratio $\frac{\sin\theta_c}{\sin\theta_s}$ is :



(A) $\frac{8}{7}$ (B) $\sqrt{\frac{8}{7}}$ (C*) $\frac{15}{14}$ (D) $\sqrt{\frac{15}{14}}$

Sol. $$\frac{g\sin\theta_c}{1+\frac{1}{2}} = \frac{g\sin\theta_s}{1+\frac{2}{5}}$$

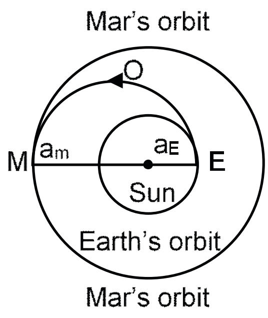
$$\frac{\sin\theta_c}{\sin\theta_s} = \frac{3/2}{7/5} = \frac{15}{14}$$

12. An n - p - n transistor has three leads A, B and C. Connecting B and C by moist fingers, A to the positive lead of an ammeter, and C to the negative lead of the ammeter, one finds large deflection. Then, A, B and C refer respectively to:

(A*) Emitter, base and collector (B) Collector, emitter and base

(C) Base, emitter and collector (D) Base, collector and emitter

13. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$ m, that of Mar's orbit $a_m = 2.28 \times 10^{11}$ m, taken Kepler's laws give the estimate of time for Mangalyan to reach Mars from Earth to be close to:



(A) 320 days (B) 500 days (C*) 260 days (D) 220 days

Sol. $r = \frac{1.5 + 2.28}{2} = 1.89$

$\frac{T_m}{T_e} = \left(\frac{1.89}{1.5}\right)^{3/2}$

$t_m = \frac{T_m}{2} = \left(\frac{1.89}{1.5}\right)^{3/2}$

$= \frac{365}{2} \times 1.41 = 257.3$ day

14. The equation of state for a gas is given by PV = nRT + αV, where n is the number of moles and α is a positive constant. The initial temperature and pressure of one mole of the gas contained in a cylinder are T_0 and P_0 respectively. The work done by the gas when its temperature doubles isobarically will be:

(A) $P_0T_0 R\ell n\, 2$ (B) $P_0T_0 R$ (C*) $\frac{P_0T_0R}{P_0 - \alpha}$ (D) $\frac{P_0T_0R}{P_0 + \alpha}$

Sol. $P_0V_0 = nRT_0$

$P_0V = nRT$

$T_f = 2T_0$

$W = \int PdV$

$= \int\left(\frac{nRT}{V} + \alpha\right)dv$

$PV = nRT + \alpha V$

$\int p\,dV = \int_{T_0}^{2T_0} nR\,dT + \int_{V_i}^{V_f} \alpha\, dv$

$= nRT_0 + \alpha\, V_i$

$$= nRT_0 + \alpha\left(\frac{nRT_0}{P_0}\right)$$

$$= nRT_0\left(1+\frac{\alpha}{P_0}\right)$$

PV = nRT + αV

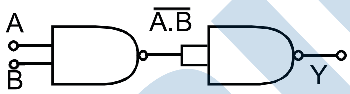
$\int$pdV = $\int$nRdT +$\int\alpha$dV

$$W = nRT_0 + \alpha\left[\frac{nRT_0}{P_0 - \alpha}\right]$$

$$= nR_0T_0\left[\frac{P_0}{P_0 - \alpha}\right]$$

$$= \frac{nRT_0\ P_0}{P_0 - \alpha}$$

15. Identify the gate and match A, B, Y in bracket to check:



(A) OR (A = 1, B = 1, Y = 0)

(B) XOR (A = 0, B = 0, Y = 0)

(C*) AND (A = 1, B = 1, Y = 1)

(D) NOT (A = 1, B = 1, Y = 1)

16. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship?

(A) a proton

(B) an α-particle

(C*) an dust particle

(D) an electron

17. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbondioxide will be close to (ℓn 5 = 1.601, ℓn 2 = 0.693).

(A) 231 s

(B) 208 s

(C) 142 s

(D*) 161 s

Sol. $8 = 10e^{-\lambda} \times 40$

$$5 = 10e^{-\frac{\lambda t}{1.3}}$$

$$\ln\frac{4}{5} = -\lambda \times 40$$

2 × 0.693 – 1.601 = – λ × 40

λ = 0.005375

$$\ln\frac{1}{2} = -\frac{\lambda t}{1.3}$$

$$-0.693 = -\frac{0.005375}{1.3}t$$

t = 167.6

18. A transverse wave is represented by: $y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$ For what value of the wavelength the wave velocity is twice the maximum particle velocity?

(A) 60 cm (B*) 40 cm (C) 10 cm (D) 20 cm

Sol. $V = 2(Vp)_{max}$

$\because V = f\lambda$

$f\lambda = 2\omega A$

$\lambda = 4\pi A$

$= 4\pi \times \frac{10}{\pi}$

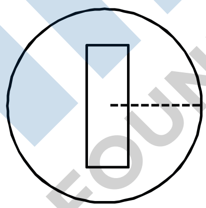
= 40 cm

19. The magnetic field of earth at the equator is approximately 4×10^{-5} T. The radius of earth is 6.4×10^{6} m. Then the dipole moment of the earth will be nearly of the order of:

(A) 10^{16} A m^2 (B*) 10^{23} A m^2 (C) 10^{20} A m^2 (D) 10^{10} A m^2

Sol. $B = 4 \times 10^{-5}$T

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{r^3} = 10^{-7} \times \frac{M}{(96.4 \times 10^6)^3} = 4 \times 10^{-5}$$



$$M = \frac{4 \times 10^{-5} \times 10^{18} \times 6.4^3}{10^{-7}}$$

$= 1.048 \times 10^{3+18+7-5}$

$= 10^{23}$

20. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about 150 N/C, directed inward towards the centre of the Earth. This gives the total net surface charge carried by the Earth to be: [Given $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2, $R_E = 6.37 \times 10^{6}$ m]

(A) – 670 kC (B) + 680 kC (C) + 670 kC (D*) – 680 kC

Sol. $E = \frac{1}{4\pi\varepsilon_0}\frac{\theta}{R^2} = \frac{\sigma}{\varepsilon_0}$ $\Rightarrow \sigma = \varepsilon_0 E$

$= 8.85 \times 10^{-12} \times 150$

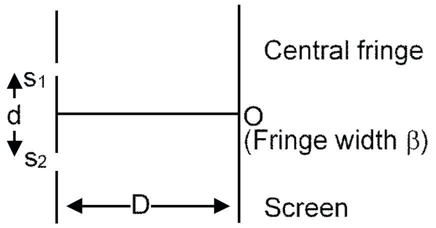
$Q = \varepsilon_0 E \times 4\pi R^2$

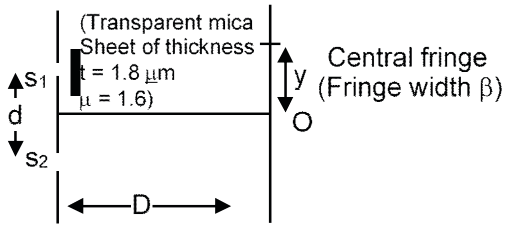
$= 6.76 \times 10^{5} \times 10^{-12} \times 10^{+12}$

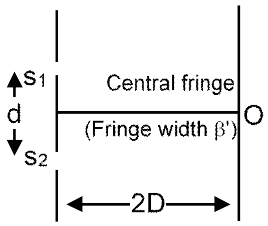
= 680 kC

for inward will be negative.

21. Using monochromatic light of wavelength λ, an experimentalist sets up the Young' double slit experiment in three ways as shown. If the observes that $y = \beta'$, the wavelength of light used is:







(A) 560 nm (B) 580 nm (C) 520 nm (D*) 540 nm

Sol. $B' = y$

$(\mu - 1)t = d\sin\theta$

$$= d\theta = \frac{dy}{D}$$

$$y = \frac{D(\mu-1)}{d}t$$

$$\frac{(2D)\lambda}{d} = \frac{D(\mu-1)}{d}t$$

$$\lambda = \frac{(\mu-1)t}{2} = \frac{(1.6-1)\times 1.8\mu m}{2}$$

$= 0.6 \times 0.9\ \mu m$

$= .54\ \mu m$

$= 540$ nm

22. Water is flowing at a speed of 1.5 ms^{-1} through a horizontal tube of cross-section area 10^{-2} m^2 and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density of water = 10^3 kgm^{-3}).

(A*) 22.5 N (B) 33.7 N (C) 45 N (D) 15 N

Sol. $F = v\frac{dm}{dt}$

$= v\ A\rho v$

$= v^2 A\rho$

$= (1.5)^2 \times 10^{-2} \times 10^3$

$= 2.25 \times 10 = 22.5$ N

23. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be:

(A) 2.7% (B*) 2.27% (C) 1.7% (D) 4.4%

Sol. $T^2 = \frac{4\pi^2\ell}{g}$

$$g = 4\pi^2 \frac{\ell}{T^2}$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta\ell}{\ell} \times 100\right) + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= \left(\frac{0.1}{20} \times 100\right) + 2\left(\frac{0.01}{.9} \times 100\right)$$

$$= 0.5 + 2 \times \frac{10}{9} = 0.5 + 2.2 = 2.7\%$$

24. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC source is 100 V, then V_R is close to :

(A) 100 V (B) 70 V (C*) 90 V (D) 50 V

Sol. $I = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{100}{\sqrt{9x^2 + x^2}} = \frac{100}{\sqrt{10x^2}}$

Since $V_L : V_C : V_R = 1 : 2 : 3$

$X_L = X_C : X_R = 1 : 2 : 3$

$= x : 2x : 3x$

now $VR = I\,(3x)$

$$= \frac{100}{\sqrt{10x^2}} \cdot 3x\ 100$$

$$\approx 94.87\ V$$

25. In materials like aluminium and copper, the correct order of magnitude of various elastic modulii is:

(A) Young's modulii < Shear modulii < Bulk modulii

(B) Bulk modulii < Shear modulii < Young's modulii

(C*) Shear modulii < Young's modulii < Bulk modulii

(D) Bulk modulii < Young's modulii < Shear modulii

26. The position of a projectile launched from the origin at t = 0 is given by $\vec{r} = (40\hat{i} + 50\hat{j})m$ at t = 2s. If the projectile was launched at an angle θ from the horizontal, then θ is (take g = 10 ms^{-2})

(A*) $\tan^{-1}\frac{7}{4}$ (B) $\tan^{-1}\frac{4}{5}$ (C) $\tan^{-1}\frac{3}{2}$ (D) $\tan^{-1}\frac{2}{3}$

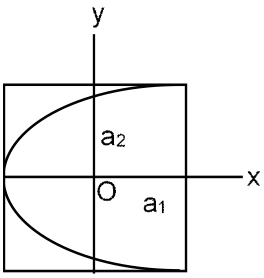
Sol. $2u_x = 40 \Rightarrow 4x = 20$

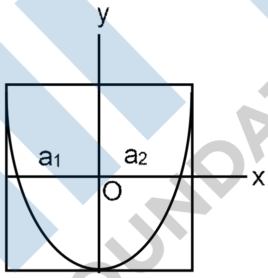
$$50 = 24y - \frac{1}{2} \times 10 \times 22 \Rightarrow 4y = 35$$

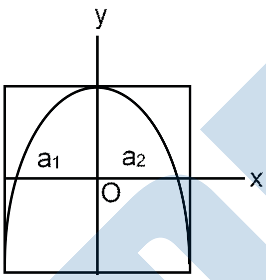
$$\tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

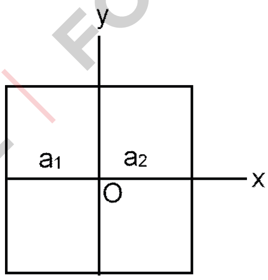
$$\theta = \tan^{-1}\left(\frac{7}{4}\right)$$

27. A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by; $x = a_1 \cos \omega t$ and $y = a_2 \cos 2\omega t$ traces a curve given by:

(A)


(B*)


(C)


(D)


Sol. $y = a_2\,[2\cos^2\omega t - 1]$

$$= a_2\left[2\cdot\frac{x^2}{a_1^2} - 1\right]$$

$$y = \frac{2a_2}{a_1^2}x^2 - a_2$$

at x = 0, y is negative and this is a equation parabola. Hence answer is 4.

28. Match List - I (Wavelength range of electromagnetic spectrum) with list II. (Method of production of these waves) and select the correct option from the options given below the lists.

	List-I		List - II
(a)	700 nm to 1 mm	(i)	Vibration of atoms and molecules

(b)	1 nm to 400 nm	(ii)	inner shell electrons in atoms moving from one energy level to a lower level
(c)	$< 10^{-3}$ nm	(iii)	Radioactive decay of the nucleus
(d)	1 mm to 0.1 m	(iv)	Magnetron valve

(A) (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i) (B) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(C) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i) (D*) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

Sol. 10^{19}

$$E = \frac{hC}{\lambda} = hV \qquad \lambda = \frac{C}{V} = \frac{10^8}{10^{19}}$$

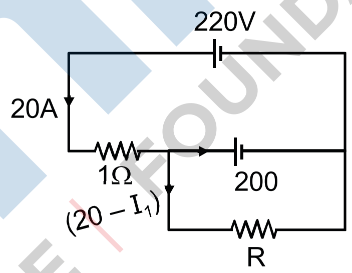
$= 10^{-11}$ m $\qquad = 10^{-2}$ nm

Magnetron valve generate microwaves.

29. A d.c. main supply of e.m.f. 220 V is connected across a storage battery of e.m.f. 200 V through a resistance of 1 Ω. The battery terminals are connected to an external resistance 'R'. The minimum value of 'R', so that a current passes through the battery to charge it is:

(A*) 11 Ω (B) 9 Ω (C) 7 Ω (D) Zero

Sol. $(20 - I_1)\,R = 200$



$$R = \frac{200}{(20 - I_1)}$$

$R \longrightarrow$ Minimum

when $20 - I_1 \longrightarrow$ maximum

& I_1 cannot be zero

so R □ 11Ω

30. The focal lengths of objective lens and eye lens of a Gallelian Telescope are respectively 30 cm and 3.0 cm. Telescope produces virtual, erect image of an object situated far away from it at least distance of distinct vision from the eye lens. In this condition, the Magnifying Power of the Gallelian Telescope should be:

(A) – 11.2 (B) + 11.2 (C) – 8.8 (D) + 8.8

Sol. $f_0 = 30$ cm $\qquad f_e = 3$cm

$$M = \frac{f_0}{f_e}\left(1 - \frac{f_C}{D}\right)$$

$$= \frac{30}{3}\left(1 - \frac{3}{25}\right)$$

$$= \frac{22 \times 30}{3 \times 25} = \frac{44}{5} = +8.8$$

PART-B : CHEMISTRY

31. The correct statement about he magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ is : (Z = 26)

(A) $[Fe(CN)_6]^{3-}$ is paramagnetic, $[FeF_6]^{3-}$ is diamagnetic.

(B) $[Fe(CN)_6]^{3-}$ is diamagnetic, $[FeF_6]^{3-}$ is paramagnetic.

(C*) both are paramagnetic.

(D) both are diamagnetic

Sol. In $[FeF_6]^{3-}$, 5 unpaired electron present is

In $[Fe(CN)_6]^{3-}$ 1 unpaired electron present.

32. Which of the following is not formed when H_2S reacts with acidic $K_2Cr_2O_7$ solution?

(A) S (B) $Cr_2(SO_4)_3$ (C) K_2SO_4 (D*) $CrSO_4$

Sol. $K_2Cr_2O_7 + H_2S \longrightarrow Cr_2(SO_4)_3 + S + K_2SO_4 + H_2O$

33. Which is the major product formed when acetone is heated with iodine and potassium hydroxide?

(A) Iodoacetone (B) Acetopenone (C*) Iodoform (D) Acetic acid

Sol. $$CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \xrightarrow[\text{Iodoform reaction}]{I_2 + KOH} \underset{\text{Iodoform}}{CHI_3} + CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-\overset{\ominus}{O}\overset{\oplus}{K}$$

34. In the following sets of reactants which two sets best exhibit the amphoteric character of $Al_2O_3.xH_2O$?

Set 1 : $Al_2O_3.xH_2O(s)$ and OH^- (aq) Set 2 : $Al_2O_3.xH_2O(s)$ and H_2O (ℓ)

Set 3 : $Al_2O_3.xH_2O(s)$ and $H^+(aq)$ Set 4 : $Al_2O_3.xH_2O(s)$ and $NH_3(aq)$

(A) 3 and 4 (B*) 1 and 3 (C) 2 and 4 (D) 1 and 2

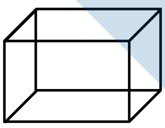
Sol. In set 1 : $Al(OH)_4^-$ is formed

In set 2 : Al^{+3} & H_2O is formed

35. In a face centered cubic lattice atoms A are at the corner points and atoms B at the face centered points. If atom B is missing from one of the face centered points, the formula of the ionic compound is:

(A) AB_2 (B*) A_2B_5 (C) A_5B_2 (D) A_2B_3

Sol.



$A = 8 \times \frac{1}{8} = 1$

$B = 6 \times \frac{1}{2} - 1 \times \frac{1}{2} = \frac{5}{2}$

A : B

$1 : \frac{5}{2} \quad \Rightarrow \quad 2 : 5$

36. Which one of the following reactions will not result in the formation of carbon-carbon bond?

(A) Wurtz reaction
(B*) Cannizzaro reaction
(C) Reimer-Tieman reaction
(D) Friedel Craft's acylation

Sol. In cannizaro reaction carbon-carbon bond not formed

$$2H\text{–}\overset{O}{\overset{\|}{C}}\text{–}H \xrightarrow{\text{Conc. KOH}} H\text{–}\overset{O}{\overset{\|}{C}}\text{–}\overset{\ominus}{O}\overset{\oplus}{K} + CH_2\text{–}OH$$

37. Which one of the following class of compounds is obtained by polymerization of acetylene?

(A) Poly-ester
(B) Poly-yne
(C) Poly-amide
(D*) Poly-ene

Sol. $\underset{\text{yne}}{nHC{\equiv}CH} \xrightarrow{\text{Polymerisation}} \underset{\text{Poly-yne}}{-\!\!(CH{=}CH)_n\!\!-}$

38. Dissolving 120 g of a compound of (mol wt. 60) in 1000 g of water gave a solution of density 1.12 g/mL. The molarity of the solution is:

(A) 2.50 M
(B) 4.00 M
(C*) 2.00 M
(D) 1.00 M

Sol. Molarity of solution $= \dfrac{\left(\dfrac{120}{60}\right)}{\dfrac{1120}{1.12} \times \dfrac{1}{1000}}$

$= 2$ M

39. The number and type of bonds in C_2^{2-} ion in CaC_2 are:

(A) Two σ bonds and one π-bond
(B) Two σ bonds and two π-bonds
(C) One σ bond and one π-bond
(D*) One σ bond and two π-bonds

Sol. $Ca^{+2}\ [C{\equiv}C]^{-2}$

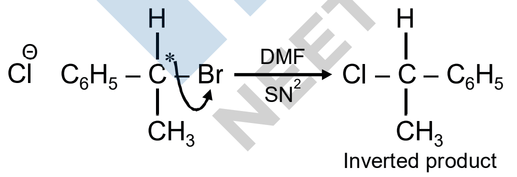
40. In a nucleophilic substitution reaction:

$R\text{–}Br + Cl^- \xrightarrow{\text{DMF}} R\text{–}Cl + Br^-$,

which one of the following undergoes complete inversion of configuration?

(A) $C_6H_5CHCH_3Br$
(B) $C_6H_5CCH_3C_6H_5Br$
(C*) $C_6H_5CH_2Br$
(D) $C_6H_5CHC_6H_5Br$

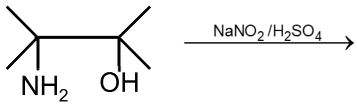
Sol.

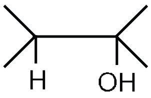
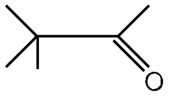
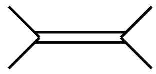
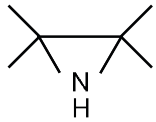


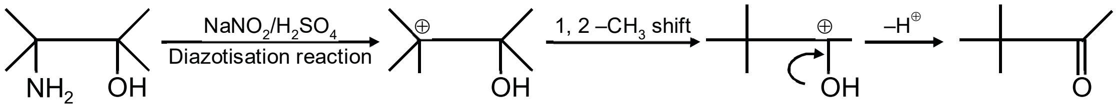
41. Which of the following has unpaired electron(s)?

(A) N_2
(B*) O_2^-
(C) O_2^{2-}
(D) N_2^{2+}

Sol. O_2^- has one unpaired electron is π^* MO.

42. The major product of the reaction  is

(A)  (B*)  (C)  (D) 

Sol. 

43. The temperature at which oxygen molecules have the same root mean square speed as helium atoms have at 300 K is:

(Atomic masses: He = 4 u, O = 16 u)

(A) 1200 K (B) 300 K (C) 600 K (D*) 2400 K

Sol. $(U_{rms})_{O_2} = (U_{rms})_{He}$

$$\Rightarrow \frac{3RT_{O_2}}{32} = \frac{3RT_{He}}{4}$$

$$\Rightarrow T_{O_2} = 8 \times 300 = 2400 \text{ K}$$

44. An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is:

(A) sp^3d^2 (B) dsp^2 (C*) d^2sp^3 (D) dsp^3d

Sol. Co^{+3} is diamagnetic & having d^6 by configuration under SFL.

45. The standard electrode potentials $\left(E^\circ_{M^+/M}\right)$ of four metals A, B, C and D are –1.2 V, 0.6 V, 0.85 V and –0.76 V, respectively. The sequence of deposition of metals on applying potential is :

(A) D, A, B, C (B) A, C, B, D (C*) C, B D, A (D) B, D, C, A

Sol. Higher the value of reduction potential more wil be the ease of deposition.

46. Vander Waal's equation for a gas is stated as, $p = \frac{nRT}{V - nb} - a\left(\frac{n}{v}\right)^2$. This equation reduces to the perfect gas equation, $p = \frac{nRT}{V}$ when,

(A) both temperature and pressure are very high.

(B*) temperature is sufficiently high and pressure is low.

(C) temperature is sufficiently low and pressure is high

(D) both temperature and pressure are very low.

47. The energy of an electron in first Bohr orbit of H–atom is –13.6 eV. The energy value of electron in the excited state of Li^{2+} is:

(A) 30.6 eV (B) 27.2 eV (C*) –30.6 eV (D) –27.2

Sol. Energy of e– in the excited state of Li^{+2}

$$E = -13.6\frac{z^2}{n^2}$$

$$= -13.6 \times \frac{(3)^2}{(2)^2}\ \text{eV}$$

$$= -\frac{9}{4} \times 13.6\ \text{eV}$$

= – 30.6 eN

48. The form of iron obtained from blast furnace is

(A*) Pig iron (B) Cast iron (C) Wrought Iron (D) Steel

Sol. Iron obtained in blast furnace is known as pig iron.

49. Allyl phenyl ether can be prepared by heating:

(A) $C_6H_5 - CH = CH - Br + CH_3 - ONa$ (B) $C_6H_5Br + CH_2 = CH - CH_2 - ONa$

(C) $CH_2 = CH - Br + C_6H_5 - CH_2 - ONa$ (D*) $CH_2 = CH - CH_2 - Br + C_6H_5ONa$

Sol. $C_6H_5\overset{\ominus\ \oplus}{ONa} + CH_2{=}CH{-}CH_2{-}Br \longrightarrow \underset{\text{Allyl phenyl ether}}{C_6H_5O{-}CH_2{-}CH{=}C_6H_5}$

50. In the hydroboration-oxidation reaction of propene with diborane, H_2O_2 and NaOH, the organic compound formed is:

(A) CH_3CH_2OH (B) $CH_3CHOHCH_3$ (C) $(CH_3)_3COH$ (D*) $CH_3CH_2CH_2OH$

Sol. $CH_3 - C = CH_2 \xrightarrow[H_2O_2/\overline{O}H]{B_2H_6/THF} CH_3 - CH_2 - \underset{\displaystyle OH}{\underset{|}{CH_2}}$

51. A current of 10.0 A flows for 2.00 h through an electrolytic cell containing a molten salt of metal X. This results in the decomposition of 0.250 mol of metal X at the cathode. The oxidation state of X in the molten salt is: (F = 96,500 C)

(A*) 3+ (B) 4+ (C) 1+ (D) 2+

Sol. $W = \frac{E}{96500} \times It$

$\Rightarrow$ No. of moles $= \frac{It}{96500 \times (n - \text{factor})}$

$\Rightarrow 0.25 = \frac{10 \times 2 \times 60 \times 60}{96500 \times n - \text{factor}}$

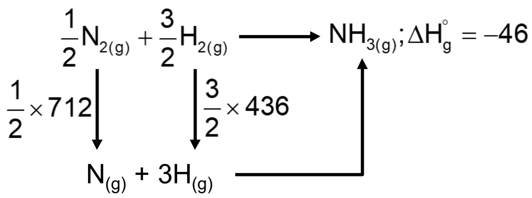
$\Rightarrow n - \text{factor} = \frac{720 \times 4}{965} = 3$

$\therefore$ Oxidation state of molten salt is +3

52. The standard enthalpy of formation of NH_3 is –46.0 kJ/mol. If the enthalpy of formation of H_2 from its atoms is –436 kJ/mol and that of N_2 is –712 kJ/mol, the average bond enthalpy of N–H bond in NH_3 is:

(A*) + 352 kJ/mol (B) –1102 kJ/mol (C) –764 kJ/mol (D) + 1056 kJ/mol

Sol. Given :



Average bond enthalpy of N–H bond = + 352 kJ mol.

53. Chloro compound of Vandium has only spin magnetic moment of 1.73 BM. This Vandium chloride has the formula:

(at. no. of V = 23)

(A) VCl_5 (B) VCl_3 (C*) VCl_4 (D) VCl_2

Sol. If the magnetic moment is 1.73 BM then the number of unpaired e^- V^{4+} having our unpaired electron

54. At a certain temperature, only 50% HI is dissociated into H_2 and I_2 at equilibrium. The equilibrium constant is:

(A) 0.5 (B) 3.0 (C) 1.0 (D*) 0.25

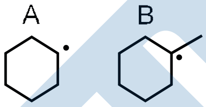
Sol. $2HI \rightleftharpoons H_2 + I_2$

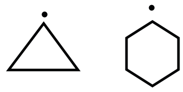
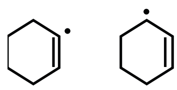
$1-\alpha \quad \frac{\alpha}{2} \quad \frac{\alpha}{2}$

$$K_{eq} = \frac{\left(\frac{\alpha}{2}\right)^2}{(1-\alpha)^2} = \frac{\alpha^2}{4(1-\alpha)^2}$$

$$K_{eq} = \frac{\left(\frac{1}{2}\right)^2}{4(1-2)^2} = \frac{1}{4}$$

55. In which of the following pairs A is more stable than B?

(A) 

(B*) $Ph_3\dot{C}$, $(CH_3)_3\dot{C}$ (C)  (D) 

Sol. $Ph_3C^\circ > (CH_3)C^\circ$

due to resonance

56. For the compounds

CH_3Cl, CH_3Br, CH_3I and CH_3F, the correct order of increasing C-halogen bond length is:

(A*) $CH_3F < CH_3Cl < CH_3Br < CH_3I$ (B) $CH_3F < CH_3I < CH_3Br < CH_3Cl$

(C) $CH_3Cl < CH_3Br < CH_3F < CH_3I$ (D) $CH_3F < CH_3Br < CH_3Cl < CH_3I$

Sol. As the radius of halogen increases C—halogen bond length incread.

57. The half-life period of a first order reaction is 15 minutes. The amount of substance left after one hour will be:

(A) 1/4 of the original amount (B) 1/8 of the original amount

(C) 1/32 of the original amount (D*) 1/16 of the original amount

Sol. Since : $t_{1/2}$ = 15 min.

$\therefore$ No. of half lives $= \dfrac{60}{15} = 4$

$\therefore$ Amount of substance left after one hour

$$= \frac{A_0}{(2)^n} = \frac{A_0}{(2)^4} = \frac{A_0}{16}$$

58. The amount of oxygen in 3.6 moles of water is:

(A) 18.4 g (B) 115.2 g (C) 28.8 g (D*) 57.6 g

Sol. 3.6 moles of H_2O = 3.6 moles of O

= 3.6 × 16 gm of oxygen

= 57.6 gm

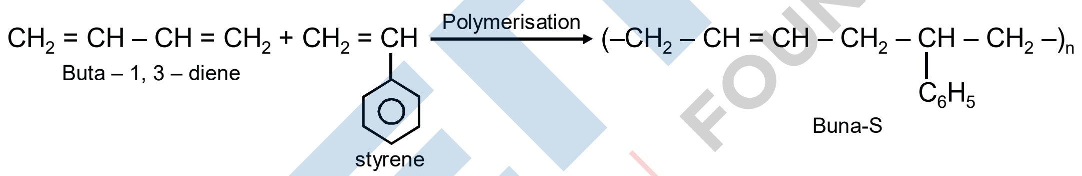
59. Structure of some important polymers are given. Which one represents Buna-S?

(A) $(-CH_2 - \overset{\overset{\displaystyle CH_3}{|}}{C} = CH - CH_2 -)_n$

(B*) $(-CH_2 - CH = CH - CH_2 - \underset{\underset{\displaystyle C_6H_5}{|}}{CH} - CH_2 -)_n$

(C) $(-CH_2 - \overset{\overset{\displaystyle Cl}{|}}{C} = CH - CH_2 -)_n$

(D) $(-CH_2 - CH = CH - CH_2 - \underset{\underset{\displaystyle CN}{|}}{CH} - CH_2 -)_n$

Sol. 

60. The gas evolved on heating CaF_2 and SiO_2 with concentrated H_2SO_4, on hydrolysis gives a white gelatinous precipitate. The precipitate is:

(A) silica gel (B) hydrofluosilicic acid

(C) calcium fluorosilicate (D*) silicic acid

Sol. $CaF_2 + H_2SO_4 \longrightarrow H_2F_2 + Ca(HSO_4)_2$

$SiO_2 + 2H_2F_2 \longrightarrow SiF_4 + 2H_2O$

$SiF_4 + H_2O \longrightarrow H_2[SiF_6]$

PART-C : MATHEMATICS

61. In a set of 2n distinct observations, each of the observation below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations

(A) increases by 1 (B) decreases by 2 (C*) increases by 2 (D) decreases by 1

Sol. $\frac{t_1+t_2+t_3.....t_n+t_{n+1}+.....+t_{2n}}{2n}=M$

$\frac{t_1+5+t_2+5+.....t_n+5+t_{n+1}-3+.....+t_{2n}-3}{2n}$

$\frac{t_1+t_2+.....t_{n-1}+5(n)+t_n+t_{n-1}+.....+t_{2n}-3(n)}{2n}$

$\frac{t_1+t_2+t_3+.....t_{2n}}{2n}+\frac{5n-3n}{2n}=M+1$

62. A line in the 3-dimensional space makes an angle θ $\left(0<\theta\le\frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of θ is the interval

(A) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ (B) $\left(0,\frac{\pi}{4}\right]$ (C) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$ (D*) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

Sol. for min, if the line lies on x y plane it makes angle of 45°

for max. If line at z-axis it makes an angle of 90°

$\Rightarrow \left[\frac{\pi}{4},\frac{\pi}{2}\right]$

63. Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is

(A) 7x + 2y + 4z = 54 (B*) 4x + 3y + 5z = 50 (C) 5x + 4y + 3z = 57 (D) 3x + 4y + 5z = 49

Sol. $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}=\alpha$

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\beta$

Solve the above equation to find the point of intersection i.e. (4, 3, 5)

e.g. of plane with dr's l, m, n as distance from origin is d is $\ell x + my + nz = d$

dr's of (4, 3, 5) joined with origin

$\left(\frac{4}{\sqrt{50}},\frac{3}{\sqrt{50}},\frac{5}{\sqrt{50}}\right)$

$\therefore$ eq of plane

$\frac{4}{\sqrt{50}}x+\frac{3}{\sqrt{50}}y+\frac{5}{\sqrt{50}}z=\sqrt{50}$

4x + 3y + 5z = 50

64. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2)\dfrac{dy}{dx} = g(x)\, y$, then g(x) equals

(A*) 2x (B) $2x^2$ (C) $\dfrac{x^2}{2}$ (D) $\dfrac{x}{2}$

Sol. $x^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2ay = 0$ ......(i)

diff. w.r.t. x

$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$$

$$a = \frac{x + y.y'}{y'} \quad ......(ii)$$

put (ii) in (i)

$$x^2 + y^2 - 2y\left(\frac{x + y.y'}{y'}\right) = 0$$

$(x^2 - y^2)y' = 2xy$ ....(iii)

compare (iii) with $(x^2 - y^2)\dfrac{dy}{dx} = g(x).y$

gives g(x) = 2x

65. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6 without repetition, is

(A) 36 (B) 432 (C) 18 (D*) 108

Hint: 3! (3 + 4 + 5 + 6).

Sol. $(6 + 5 + 4 + 3)\,\underline{|3}$

= 18 × 6

= 108

66. Let P be the relation defined on the set of all real numbers such that

P = {(a, b) : $\sec^2 a - \tan^2 b = 1$}. Then P is

(A) reflexive and symmetric but not transitive
(B) symmetric and transitive but not reflexive
(C*) an equivalence relation
(D) reflexive and transitive but not symmetric

Sol. for reflexive : $\sec^2 a - \tan^2 a = 1$ an identity for all

$x \in R \Rightarrow$ reflexive

for symmetric : sec2a – tan2b = 1 ...(i) to prove

$\sec^2 b - \tan^2 a = 1$

$\sec^2 b - \tan^2 a = 1 + \tan^2 b - (\sec^2 a - 1) = 1 + \tan^2 b + 1 - \sec^2 a = \sec^2 a - \tan^2 b = 1 \Rightarrow$ symmetric

[$\because$ from (1)]

for transitive :

$\sec^2 a - \tan^2 b = 1$ ...... (ii)

$\sec^2 b - \tan^2 c = 1$ ......(iii)

to prove : $\sec^2 a - \tan^2 c = 1$

proof L.H.S.

$1 + \tan^2 b + 1 - \sec^2 b$ from (ii) & (iii)

$= \sec^2 b - \tan^2 b$ identity

$= 1$

$\Rightarrow$ P is reflexive, symmetric and transitive.

67. Given three points P, Q, R with P(5, 3) and R lies on the x-axis. If equation of RQ is $x - 2y = 2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line

(A) $x - 2y + 1 = 0$ (B) $2x + y - 9 = 0$ (C) $5x - 2y = 0$ (D*) $2x - 5y = 0$

Sol. equation of RQ $\equiv x - 2y = 2$

$\Rightarrow$ R (2, 0)

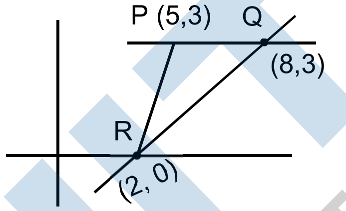
equation of PQ = y = 3

point of intersection of PQ and RQ

$x - 2(3) = 2$

$x = 8$

$\Rightarrow$ R (8, 3)

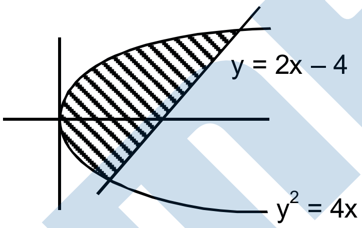


Centroid $\left(\frac{2+8+5}{3}, \frac{0+3+3}{3}\right)$

68. Let A = {(x, y): $y^2 \le 4x$, $y - 2x \ge -4$}. The area (in square units) of the region A is

(A) 8 (B) 10 (C) 11 (D*) 9

Sol.



solve for y; $y^2 = 4x$ as $y - 2x = -4$

gives $y = -2, 4$

$\Rightarrow$ Area $= \int_{-2}^{4}\left(\frac{y+4}{2} - \frac{y^2}{4}\right)dy = \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^{4}$

$= 9$

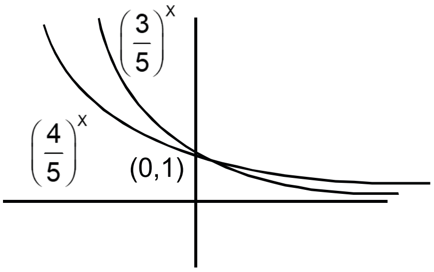
69. If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation $f(x) = 0$ has

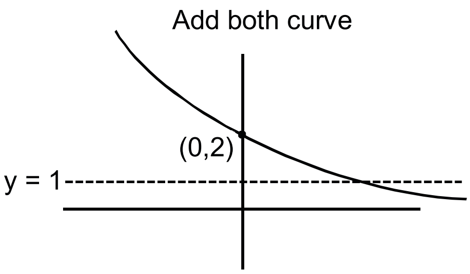
(A) more than two solutions (B) no solution

(C) two solutions (D*) one solution

Hint: $x = 2$

Sol.



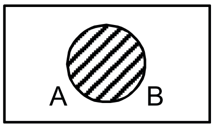


70. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the incorrect statement amongst the following statements is

(A*) $P(A) + P(B) = 1$ (B) $P(A' \cap B) = 0$

(C) A and B are equally likely (D) $P(A \cap B') = 0$

Sol.



$\Rightarrow A = B$

71. The contrapositive of the statement "I go to school if it does not rain" is

(A) if I go to school, it rains (B) if it rains, I do not go to school

(C*) if I do not go to school, it rains (D) if it rains, I go to school

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

72. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{4}$ (D*) $\frac{1}{2}$

Sol. $a^2 = b^2 + c^2$ ....(i)

given $a^2 + a^2 = (2c)^2$

$2a^2 = 4c^2$

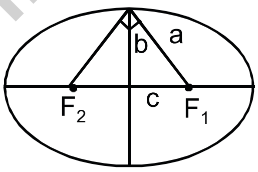
$a^2 = 2c^2$ ....(ii)

$c = ae$

$\frac{c}{a} = e$

$\frac{c^2}{a^2} = e^2$

$\frac{c^2}{2c^2} = e^2 \quad \Rightarrow \quad e^2 = \frac{1}{2}$



73. If $y = e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to

(A) 1 (B) $n\,e^{-nx}$ (C) $n\,e^{nx}$ (D*) $-n\,e^{-nx}$

Sol. $y = e^{nx}$

$\frac{1}{n}\log y = x$

$\frac{dy}{dx} = ne^{nx}$ $\qquad \frac{1}{n}\left(\frac{1}{y}\right) = \frac{dx}{dy}$

$\frac{d^2y}{dx^2} = n^2e^{nx}$ ......(i) $\qquad -\frac{1}{ny^2} = \frac{d^2x}{dy^2}$ .....(ii)

(i) × (ii) = $n^2e^{nx} \cdot \frac{1}{ny^2} = \frac{n^2y}{ny^2} = \frac{n}{y} = \frac{n}{e^{nx}}$

74. If cosec $\theta = \frac{p+q}{p-q}$ θ = $(p \neq q \neq 0)$, then $\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right|$ is equal to

(A) $\sqrt{\frac{p}{q}}$ (B*) $\sqrt{\frac{q}{p}}$ (C) $\sqrt{pq}$ (D) pq

Sol. $\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right| = \left|\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right|$

$= \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}} \times \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}$

$= \frac{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1 - \frac{p-q}{p+q}}{\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}} = \frac{\sqrt{q}}{\sqrt{p}}$

75. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is

(A) 6 (B*) 3 (C) 1 (D) 4

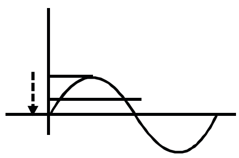
Hint: $(\sin\alpha - 1)[(2 - \sin\alpha - 1)(\sin\alpha - 2)] = 0$.

Sol. $2\sin^3\alpha - 2 = 7\sin^2\alpha - 7\sin\alpha$

$2(\sin\alpha - 1)(\sin^2\alpha + 1 + \sin\alpha) = 7\sin\alpha(\sin\alpha - 1)$

$\Rightarrow \sin\alpha = 1$ or $2\sin^2\alpha + 2 + 2\sin\alpha = 7\sin\alpha(\sin\alpha - t)(\sin\alpha - 2)$

$\Rightarrow \sin\alpha = 1$ or $\sin\alpha = \frac{1}{2}$ $\qquad \because \sin\alpha \neq -2$



$\Rightarrow$ 3 solutions

76. If a, b, c are non-zero real numbers and if the system of equations (a – 1) x = y + z, (b – 1) y = z + x, (c – 1) z = x + y,

has a non-trivial solution, then (ab + bc + ca) equals

(A*) abc (B) – 1 (C) a + b + c (D) 1

Sol. for non-trivial solution D = 0

$$\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$R_1 \to R_1 - R_3 \quad \begin{vmatrix} -a & 0 & c \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix}$$

$\Rightarrow$ a {(1–b)(1–c)–1} + c{1–(1–b)}=0

$\Rightarrow$ ab + ac + bc – abc = 0

$\Rightarrow$ ab + ac + bc = abc

77. If f(x) is continuous and $f\left(\frac{9}{2}\right) = \frac{2}{9}$, then $\lim\limits_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to

(A) $\frac{8}{9}$ (B) $\frac{9}{2}$ (C) 0 (D*) $\frac{2}{9}$

Sol. $f\left(\frac{2\sin^2\frac{3x}{2}}{\frac{4}{9}\cdot\frac{3x}{2}\cdot\frac{3x}{2}}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}$

78. If B is a 3 × 3 matrix such that $B^2 = 0$, then det. $[(I + B)^{50} – 50B]$ is equal to

(A) 3 (B) 50 (C) 2 (D*) 1

Sol. $[(1 + B)^{50} – 50\,B] = 1 + 50B + \frac{50.49}{\underline{|2}}B^2 + \ldots\ldots - 50B$

= 1 + B2 {-----------} = 1 + 0 {-----} = 1

79. The number of terms in the expansion of $(1 + x)^{101} (1 + x^2 – x)^{100}$ in powers of x is

(A*) 202 (B) 101 (C) 302 (D) 301

Sol. $(1 + x) (1 +x)^{100} (1 + x^2 – x)^{100} = (1 + x) (1 + x^3)^{100}$

$$= \underbrace{1(1+x^3)^{100}}_{101\,\text{terms}} + \underbrace{x(1+x^3)^{100}}_{101\,\text{terms}}$$

and no term is of same exponent of x

$\Rightarrow$ 202 terms

80. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0$ ($a \neq 0$, $a, b \in R$), then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ has roots

(A*) $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$ (B) $\alpha^{\frac{-3}{2}}$ and $\beta^{\frac{-3}{2}}$ (C) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (D) $\alpha\beta^{\frac{1}{2}}$ and $\alpha^{\frac{1}{2}}\beta$

Sol. $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = -\frac{b}{a}$ also $\frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a} \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = -b$

now $x(x + b^3) + a^3 - 3abx$

$= x^2 + (b^3 - 3ab)x + a^3 = x^2 + b(b^2 - 3a)x + a^3$

$= x^2 - \left(\sqrt{\alpha} + \sqrt{\beta}\right)\left\{\alpha + \beta + \sqrt{\alpha\beta} - 3\sqrt{\alpha\beta}\right\}x + \alpha\beta\sqrt{\alpha\beta}$

$= x^2 - \left(\alpha\sqrt{\alpha} + \beta\sqrt{\beta}\right) + \alpha\beta\sqrt{\alpha\beta}$

$\Rightarrow$ roots are $\alpha\sqrt{\alpha}$ and $\beta\sqrt{\beta}$

81. The integral $\int_0^{\frac{1}{2}} \frac{\ell n(1+2x)}{1+4x^2}dx$, equals

(A) $\frac{\pi \ell n 2}{8}$ (B) $\frac{\pi \ell n 2}{4}$ (C) $\frac{\pi \ell n 2}{32}$ (D*) $\frac{\pi \ell n 2}{16}$

Sol. $\int_0^{1/2} \frac{\ell n(1+2x)}{1+(2x)^2}dx$ Put $2x = \tan\theta$

$dx = \frac{1}{2}\sec^2\theta\, d\theta$

at $x = 0$, $\theta = 0$, at $x = \frac{1}{2}, \theta = \frac{\pi}{4}$

$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2\theta}, \frac{1}{2}\sec^2\theta\, d\theta$

$I = \frac{1}{2}\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta, \frac{1}{2} I_1$

$I = \int_0^{\pi/4} \log[1+\tan\left(\frac{\pi}{4}-\theta\right)$ using property

$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right] = \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$

$I_1 = \frac{\pi}{4}\log 2 - I_1$

$I_1 = \frac{\pi}{8}\ln 2$

$\Rightarrow I = \frac{\pi}{16}\ln 2$

82. If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}$ + ...+ upto 20 terms is equal to $\frac{k}{21}$, then k is equal to

(A) 240 (B) 180 (C) 60 (D*) 120

Sol. $t_n=\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}}=\frac{6}{n(n+1)}=6\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$S_n=6\left\{\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}+......-\frac{1}{21}\right\}=6\left(\frac{1}{1}-\frac{1}{21}\right)$

$=6\left(\frac{20}{21}\right)=\frac{120}{21}$ $\Rightarrow$ k = 120

83. Let w (Im w $\neq$ 0) be a complex number. Then the set of all complex numbers z satisfying the equation w – $\bar{w}$ z = k (1 – z), for some real number k, is

(A) {z: | z | = 1} (B) {z: z = $\bar{z}$} (C) {z: z $\neq$ 1} (D*) {z: | z | = 1, z $\neq$ 1}

Sol. $w-\bar{w}z=k-kz$

$kz-\bar{w}z=k-w$

$z=\frac{k-w}{k-\bar{w}}$ ....(i)

$\bar{z}=\frac{k-\bar{w}}{k-w}$ ....(ii)

(i) × (ii)

$z\bar{z}=1$

|z| = 1

but z $\neq$ 1

84. If $|\vec{a}|$ = 2, $|\vec{b}|$ = 3 and $|2\vec{a}-\vec{b}|$ = 5, then equals

(A) 17 (B*) 5 (C) 7 (D) 1

Sol. $|2\vec{a}-\vec{b}|^2=25$

$4|\vec{a}|+|\vec{b}|^2-4\cdot\vec{a}\cdot\vec{b}=25$

$16+9-4\cdot\vec{a}\cdot\vec{b}=25$

$4\times\vec{a}\cdot\vec{b}=0$ ....(i)

now

$|2\vec{a}+\vec{b}|=k$

$(2\vec{a}+\vec{b})(2\vec{a}+\vec{b})=k^2$

$4|\vec{a}|^2+|\vec{b}|^2+4\vec{a}\cdot\vec{b}=k^2$

$\sqrt{16+9+0}=k$

5 = k

85. If equations $ax^2 + bx + c = 0$, $(a, b, c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a: b : c equals

(A) 4: 3: 2 (B) 1: 2: 3 (C) 3: 2: 1 (D*) 2: 3: 4

Sol. $2x^2 + 3x + 4 = 0$ as $D \leq 0$

$\Rightarrow$ both roots are imaginary $\Rightarrow$ both roots are common

$$\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$$

86. If the Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ in the interval $[-1, 1]$ for the point $c = \frac{1}{2}$, then the value of $(2a + b)$ is

(A) 2 (B) 1 (C) – 2 (D*) – 1

Sol. $f(-1) = -2 + a - b$, $f(1) = 2 + a + b$

$f(-1) = f(1) \Rightarrow -2 + a - b = 2 + a + b \; -2 = b$

$f'(x) = 6x^2 + 2ax + b$

$$f'\left(\frac{1}{2}\right) = 6 \cdot \frac{1}{4} + 2 \cdot a \cdot \frac{1}{2} + b = 0$$

$$\Rightarrow \frac{3}{2} + a + b = 0 \quad (\because b = -2)$$

$$\Rightarrow a = \frac{1}{2} \qquad \because 2a + b = -1$$

87. $\int \frac{\sin^8 x - \cos^8 x}{(1 - 2\sin^2 x \cos^2 x)} dx$ is equal to

(A*) $\frac{-1}{2}$ sin 2x + C (B) – $\sin^2 x$ + C (C) $\frac{-1}{2}$ sin x + C (D) sin 2x + C

Sol. $$I = \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{\{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x\}}$$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)}{(\sin^4 x + \cos^4 x)}$$

$= \int - \cos 2x$

$$= -\frac{\sin 2x}{2} + 2$$

88. Let a and b be any two numbers satisfying $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{a} + \frac{y}{b} = 1$, lies on

(A*) a circle of radius = 2 (B) a circle of radius = $\sqrt{2}$

(C) a hyperbola with each semi-axis = 2 (D) a hyperbola with each semi-axis = $\sqrt{2}$.

Sol. $m_1m_2 = -1$; bk = ah and (h, k) satisfies line bh + ak = ab

Solve, $h = \dfrac{ab^2}{a^2+b^2}$; $k = \dfrac{a^2b}{a^2+b^2}$

square & add.

89. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4^{th} term is

(A) 8 (B*) 20 (C) 24 (D) 16

Sol. (12 – d) + 12 + (12 + d) + (12 + 2d) +.... 12 + 7d

= 12 × 9 + 27d = 108 + 27d

now according to question

200 < 108 + 27d < 220

92 < 27d < 112

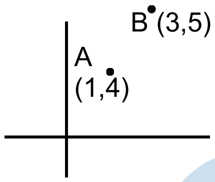
$\dfrac{92}{27} < d < \dfrac{112}{27}$ $\Rightarrow$ d = 4 only integer

$\Rightarrow$ 4th term = 12 + 2d = 12 + 8 = 20

90. If the point (1, 4) lies inside the circle $x^2 + y^2 - 6x - 10y + p = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval

(A) (0, 25) (B*) (25, 29) (C) (25, 39) (D) (9, 25)

Sol.



$AB = \sqrt{2^2+1} = \sqrt{5}$

according to question

$\sqrt{5} < \sqrt{3^2+5^2-p} < q$

5 < 34 – p < q

– 29 < – P < – 25

29 > p > 25